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Inferring Term Rates from SOFR Futures Prices*

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Abstract

The Alternative Reference Rate Committee, a group of private-sector market participants convened by the Federal Reserve, has recommended that markets transition to the use of the Secured Overnight Financing Rate (SOFR) in financial contracts that currently reference US dollar LIBOR. This paper examines the feasibility of using SOFR futures prices to construct forward-looking term reference rates that are conceptually similar to the term LIBOR rates commonly used in loan contracts. We show that futures-implied term SOFR rates have closely tracked federal funds OIS rates over the eight months since SOFR futures began trading. To examine the performance of our approach over a longer time horizon, we compare term rates derived from federal funds futures with observed overnight rates and OIS rates from 2000 to the present. Consistent with prior research, we find that futures-implied term rates accurately predict realized compounded overnight rates during most periods.

Note: This paper presents indicative forward-looking term rates derived from end-of-day SOFR futures prices. These rates are presented for informational purposes only and are not appropriate for use as reference rates in financial contracts. The rates and the process by which they were calculated do not comply with the data quality, methodology, governance and other principles for financial benchmarks established by the International Organization of Securities Commissions. These rates may differ materially from any forward-looking or backward-looking SOFR term rate that may be produced in the future by any administrator, including any such rate that may be endorsed by the Alternative Reference Rates Committee.

JEL: G12, G18
Keywords: interest rates, futures, reference rates, financial contracts, LIBOR, SOFR

1 Introduction

Structural changes in interbank funding markets since the global financial crisis have led financial regulators and market participants to question the long-term viability of US dollar LIBOR, a benchmark rate referenced by an estimated $8 trillion in business and consumer credit products and $190 trillion in derivatives.[1] In 2017 the Alternative Reference Rate Committee (ARRC), a group of private-sector market participants convened by the Federal Reserve with support from other US financial regulators, selected the Secured Overnight Financing Rate (SOFR) as the recommended

*We thank Erfan Danesh, David Bowman, and participants in the Alternative Reference Rate Committee's Term Rate Subgroup for valuable feedback. Any errors are our own. The views expressed here are those of the authors and do not reflect the views of the Federal Reserve Board or the Alternative Reference Rate Committee.

[1]Figures are notional amounts outstanding as of the end of 2016 compiled by the Alternative Reference Rate Committee (2018, Table 1).

replacement for US dollar LIBOR. Unlike LIBOR, which is reported daily for a variety of tenors ranging from overnight to one year, SOFR is an overnight rate rather than a term rate, and hence some adjustments will need to be made to contracts and systems designed to incorporate term rates.

Adapting new interest rate derivatives to reference SOFR rather than LIBOR should be relatively straightforward, since participants in derivatives markets already have substantial experience with overnight index swaps (OIS) referencing overnight rates such as the effective federal funds rate. However, the Financial Stability Board (2018) has recognized that for certain cash products that currently reference term interbank funding rates, new term rates derived from liquid markets may be needed. For many loan products tied to term LIBOR, it may be possible to use term rates derived from backward-looking compound or linear averages of observed SOFR rates. These types of averaged rates are less volatile than overnight rates, can be readily calculated from published data, and do not depend on the liquidity of any underlying market save the overnight repo market that underpins SOFR itself. In other cases, such as some business loans, borrowers with systems that cannot be easily adapted to backward-looking rates may prefer to use forward-looking term rates based on SOFR that are conceptually more similar to the term LIBOR rates they currently use.

All forward-looking term rates depend on some measure of market participants' expectations of the future path of overnight interest rates. The problems associated with term LIBOR rates stem largely from the fact that they rely on pricing information in the not particularly deep or liquid interbank wholesale funding markets and expert judgment to infer forward rates. LIBOR panel banks are required to estimate their wholesale unsecured funding costs using transaction prices if enough qualified transactions are available, but may implement judgmental approaches when transactions data are insufficient. The extent to which panel banks must rely on judgmental approaches varies depending on the tenor of the submission. According to the most recent data published by the ICE Benchmark Administration (2018), during the week of April 18, 2018 between three-fifths and four-fifths of US dollar term LIBOR submissions relied on judgmental methods.

An alternative to this approach is to rely instead on pricing information from derivatives that reference overnight rates. Although SOFR derivatives markets have just begun to develop, both the CME and ICE now offer SOFR futures contracts and there are already more transactions underlying SOFR futures than are estimated to underlie LIBOR. The use of derivatives prices to infer forward interest rates is a common practice that is well understood by market participants. Further, trading volume on SOFR derivatives markets seems likely to continue to grow at a rapid pace.

This paper examines the feasibility of computing forward-looking term reference rates based on SOFR futures contracts. The approach developed here is similar to that described by the Alternative Reference Rate Committee (2018, Appendix 3). Prices from futures contracts that reference SOFR are used to estimate market-implied forward SOFR rates at a given point in time. These forward rates are then compounded to produce forward-looking term rates. While the analysis presented here relies solely on closing prices from CME SOFR futures contracts, the methodology proposed is quite general, and could easily be adapted to incorporate data from a variety of SOFR-based derivatives contracts including futures traded on ICE or other exchanges and SOFR OIS.

Although the growth in SOFR futures has been impressive and there is considerable headroom for

liquidity to continue to improve as markets transition away from LIBOR-based products, at current levels of liquidity it is not possible to create a robust, forward-looking term rate based on intra-day SOFR futures prices. The term rates presented here are derived from end-of-day futures prices, and, as such, would not be appropriate reference rates in commercial contracts. Nonetheless, they provide a good indication of how futures-implied SOFR term reference rates would likely perform in practice.

The paper is organized as follows. Section 2 provides a brief description of our empirical approach and Section 3 presents estimated term rates derived from closing prices on CME SOFR futures. We show that our estimated term SOFR rates have closely tracked federal funds OIS rates over the eight months since SOFR futures began trading. To examine the performance of futures-implied term rates over a longer time horizon, Section 4 presents estimates of term rates using federal funds futures from 2000 to the present. Consistent with prior research, we find that futures-implied term rates are good predictors of realized compounded overnight rates during most periods, though, like other forward-looking term rates including OIS and LIBOR, they were slow to adjust to rapidly falling overnight rates during the first months of the financial crisis. Section 5 summarizes our conclusions and discusses potential extensions and improvements to the approach developed here. Technical details of our approach are described in Appendix A.

2 Inferring forward-looking term rates

Term rates can be computed from overnight rates such as SOFR by applying a simple geometric compounding formula. The difference between backward-looking and forward-looking rates lies in whether observed overnight rates or expected future overnight rates (i.e., expected forward rates) are used. Forward-looking term rates are considerably more difficult to estimate because they require that one infer market expectations from a limited set of available information. Invariably, such inference involves imposing some assumptions that restrict the shape of the path of forward rates.

In our model, expected forward SOFR rates are assumed to potentially jump up or down on scheduled Federal Open Market Committee (FOMC) policy rate announcement dates but remain constant during the periods between FOMC meetings. This assumption greatly reduces the complexity of the estimation problem by allowing one to pin down expected forward rate paths by estimating the size of rate changes on a relatively small number of fixed calendar dates.[2] Although several alternative functional forms for the forward rate path have been applied in the literature, all such approaches necessarily require that one impose some simplifying assumptions because available derivatives prices do not provide enough information to identify forward rates with daily granularity.

Figure 1, which plots SOFR rates over time, provides some justification for our modeling assumption. While realized SOFR rates do fluctuate from day to day, they tend to move within narrow bands at most times, but may jump significantly on FOMC rate announcement dates. Furthermore, daily fluctuations in SOFR rates do not appear to be very predictable, so it seems reasonable to

[2]This type of assumption is fairly common in applied models of federal funds futures and OIS. See, for example, Zucker (2010).

assume that market expectations do not typically reflect forecasts of such fluctuations.[3]

As described in detail in Appendix A, for a given estimation date, the size and direction of forward rate jumps on future FOMC announcement dates are estimated by choosing values such that the implied prices of derivatives whose settlement values depend on the forward rate path match observed prices as closely as possible. Generally speaking, in order to estimate forward rates out to a specific horizon one needs to observe prices for several contracts whose settlement windows collectively span the time horizon. We rely on closing prices for CME futures contracts for this purpose, though other derivatives instruments may also be suitable. CME Group began listing one-month and three-month SOFR futures in May 2018. One-month SOFR futures contracts are settled based on the average of daily SOFR rates during the contract delivery month and are offered for the nearest seven calendar months. Three-month SOFR futures contracts are settled based on compounded SOFR rates during the contract reference quarter and are offered in the nearest 20 quarters where quarters end in March, June, September and December.[4]

The accuracy of term rates derived from futures prices depends critically on the extent to which contract prices accurately capture market participants' expectations about forward rates. Ideally, prices should be timely, and should not embed distortions that can arise when markets are illiquid. As Figure 2 shows, liquidity in the CME SOFR futures market has been building over time, particularly for three-month contracts. Trading volume for one-month futures is concentrated in near dated contracts while three-month contracts are often used to take positions on SOFR rates at a one-year horizon (Figure 3). In this analysis, we estimate term rates out to a six month horizon, where both one-month and three-month contract prices provide information about market expectations. End-of-day futures prices are used throughout this analysis, but the methodology could accommodate use of intra-day prices instead, given sufficient trading volume.

3 Estimated SOFR term rates

To illustrate how the model functions, Figure 4 compares the estimated forward rate path for SOFR as of August 10, 2018 and September 10, 2018 with the effective federal funds rates that were observed later in the year. Jumps in the plotted forward rate paths correspond to FOMC announcement dates. Typically, forward rates will not match observed overnight rates, both because they embed a modest risk premium and because new information may arrive between the forecast date and the dates on which rates are realized. On August 10, SOFR futures prices implied about an 80 percent chance of a 25 basis point rate increase at the September 19 FOMC meeting, about a 50 percent chance of an increase at the December 12 meeting, and much lower probabilities of increases at other meetings. The anticipated September and December rate hikes did indeed occur. The differences between the predicted SOFR rates and realized EFFR rates in the top panel primarily reflects the fact that as of August 10 the rate increases were viewed by market participants as likely, but not certain. As

[3]SOFR experienced a large, but transitory, upward spike at the end of 2018 which appears to be related to the impact of year-end balance sheet adjustments and Treasury coupon settlements on a broad range of repo financing rates. Should such spikes recur on a regular basis, it might be appropriate to account for them in modeling forward SOFR rates.

[4]For detailed contract specifications, see CME Group (2018).

can be seen in the bottom panel, by September 10, when market participants had access to better information, futures-implied forward SOFR rates moved closer to those EFFR rates that actually occurred.

Forward-looking term rates are computed by compounding estimated forward overnight rates like those shown in Figure 4. Figure 5 compares estimated forward-looking term rates with overnight SOFR rates for each trading day from June 10, 2018 to January 22, 2019.[5] The large, persistent jumps in the SOFR rate correspond to the FOMC's decision to raise its policy range in September and December. These rate hikes were largely anticipated by futures markets and, accordingly, term rates moved up in advance of the policy decisions.

Much of the day-to-day fluctuations in overnight SOFR rates appears to reflect idiosyncratic factors that largely average out over time. As a result, term rates tend to be considerably less volatile than the overnight rate. In particular, it is notable that the estimated term rates were largely unaffected by a large, transitory spike in the SOFR rate at the end of December 2018 arising from financial institutions' year-end balance sheet adjustments and other factors.

Although SOFR futures markets are still relatively nascent, our modeling approach produces well behaved term rates that closely track those implied by pricing in the much more well developed federal funds OIS market. As shown in Table 1 and Figure 6, SOFR term rates typically trade within a couple of basis points of federal funds OIS rates. During our sample period, SOFR term rates were, on average, about two basis point above comparable OIS rates and more than 90 percent of observed differences were less than five basis points. The daily spread between SOFR term rates and federal funds OIS rates is typically considerably smaller than that between the overnight SOFR rate and the federal funds rate.

4 Long-run performance of futures-implied term rates

While the preceding results are encouraging, the limited time frame over which SOFR futures have been trading makes it impossible to directly assess the performance of our methodology for estimating term rates over a broad range of interest rate environments. In this section we apply the same methodology described in Section 2 to estimate term rates based on federal funds futures prices since 2000. Since, as shown in Figure 1, SOFR rates generally lie quite close to the effective federal funds rate (EFFR), term rates based on federal funds futures provide a good proxy for those based on SOFR futures. Examining the performance of these rates over the last two decades can provide insights into how SOFR term rates based on futures prices would likely perform in both very low and very high and volatile policy rate environments.

4.1 Do futures-implied term rates predict realized overnight rates?

One way to evaluate the performance of futures-implied term rates is to assess whether they provide accurate predictions of those rates that actually occurred. Because futures prices should embed

[5]SOFR futures began trading on May 7, 2018, but the first one-month and three-month contracts did not begin their settlement calculation windows until June 1 and June 20, respectively.

some risk premiums to compensate investors, there may be systematic difference in term rates and compounded realized overnight rates over time. Longstaff (2000) finds little evidence of significant term premiums in short-term repo rates while Krueger & Kuttner (1996) find that federal funds futures provide informationally efficient forecasts of effective federal funds rates that embed stable risk premiums. However, more recent research by Piazzesi & Swanson (2008) identifies meaningful time-varying, counter-cyclical expected excess returns in federal funds futures prices at horizons from one to six months. In a comparison of the predictive efficiency of federal funds futures and federal funds OIS, Lloyd (2018) concludes that both types of derivatives have similar power in predicting effective federal funds rates out to 11 months ahead and embed modest risk premiums that increase with the forecast horizon.

Table 2 and Figure 7 compare forward-looking term rates estimated from federal funds futures and realized compounded effective federal funds rates. For example, the one-month term rate on September 10, 2018 is compared to the compounded federal funds rate from September 11 to October 10. Panel (A) of Table 2 shows that overall futures-implied term rates generally do a very good job predicting realized rates. The mean difference between shorter dated term rates and realized rates is smaller than that for longer-dated rates, indicating that, as expected, shorter-dated term rates embed somewhat smaller risk premiums.

By far the largest differences between term rates and realized compounded rates occurred during the onset of the financial crisis when futures prices failed to anticipate a sequence of repeated FOMC policy rate cuts, some of which occurred between scheduled FOMC meetings. This pattern is consistent with evidence from Piazzesi & Swanson (2008) suggesting that time-varying risk premiums in federal funds markets cause futures prices to adjust sluggishly to shifts in the direction of monetary policy. It is important to note, however, that futures-implied term rates were not the only forward-looking rates to adjust slowly during this period. Figure 8 compares three-month futures-implied term rates, OIS rates, and LIBOR with realized compounded three-month effective federal funds rates during the financial crisis. Like the futures-implied rate, the federal funds OIS rate also failed to predict a fall in the policy rate, and three-month LIBOR lagged even farther behind. While some of the slow pace of adjustment of LIBOR undoubtedly reflects a contemporaneous increase in interbank counterparty risk, LIBOR panel participants do not appear to have had any better information about future federal funds rates than participants in derivatives markets.

4.2 How do futures-implied term rates compare with OIS rates?

Given the widespread application of OIS to take positions on forward term financing rates in a variety of currencies and markets, it is useful to consider whether futures-implied term rates are broadly consistent with those implied by swaps. Results presented in Section 3 demonstrate that during the second half of 2018 futures-implied SOFR term rates track federal funds OIS quite closely. In this section, we compare futures-implied term rates and OIS rates based on the effective federal funds rate over a longer time period.

Figure 9 shows term rates derived from federal funds futures and federal funds OIS over time and Figure 10 shows differences between futures- and swaps-implied rates. Differences between

federal funds futures- and swaps-implied term rates should generally be small, since both types of derivatives reference the same overnight rate. However, persistent differences could arise either because transaction costs or other frictions prevent no-arbitrage relationships from holding across futures and swaps markets or because of the presence of bilateral counterparty risk in the swaps market that is not present in the fully cleared futures market.[6] Transitory differences in rates are more likely to reflect differences in the timing of transactions throughout the trading day, which are not accounted for in the end-of-day prices used in this analysis. Typically the effects of such timing differences will be small, but they can become more pronounced when intra-day price volatility is high.

Panel (A) of Table 3 reports summary statistics on the gap between futures- and swaps-implied federal funds term rates over time. On most days, futures- and swaps-implied term rates move together. Mean spreads for three- and six-month rates are a fraction of a basis point, while one-month OIS rates are about 1.5 basis points higher, on average, than one-month futures-implied rates. Three- and six-month futures-implied term rates track OIS rates more closely than one-month rates. On more than 90 percent of days, futures-implied three- and six-month spreads differ from OIS rates by less than 3 basis points, while spreads for one-month rates covered a much wider range.

Panel (C) of Table 3 shows that the largest spreads between futures- and swaps-implied term rates occurred during the financial crisis when counterparty risk associated with uncleared swaps transactions and other market disruptions, as well as high-intra day price volatility, may have been significant factors. For example, the single largest difference between one-month term rates in our sample occurred on October 21, 2008, the date on which the Federal Reserve announced the creation of an unprecedented liquidity facility for money market investors. During the period of very low interest rates since the financial crisis, spreads between futures-implied federal funds term rates and federal funds OIS rates have been very small (panel (D) of Table 3).

5 Conclusion

This paper describes a simple approach to estimating forward-looking term rates tied to SOFR. Results using SOFR futures price data from the second half of 2018 are encouraging. Implied term SOFR term rates are similar to those derived from federal funds OIS and smoothly transition upward in advance of anticipated policy rate increases. Application of our modeling approach to federal funds futures data demonstrates that futures-implied term rates perform similarly to OIS rates over a broad range of interest rate environments.

As SOFR derivatives markets continue to evolve, a number of possible extensions to the approach described here may be possible. While only CME contract prices are used in this analysis, the methodology could readily accommodate integration of prices from additional futures contracts offered on ICE or other exchanges or even over-the-counter SOFR swaps. In estimating forward rates, our model weights all futures prices equally. If a broader set of derivatives contracts were used, one might consider weighting contracts by measures of market depth or trading volume. This

[6] Today, rigorous margin requirements and central clearing substantially mitigates bilateral counterparty risk in federal funds OIS, but these measures were less prevalent earlier in our study period.

analysis relies on end-of-day futures prices, but as market depth builds it would be desirable to use a narrower trading window to ensure that prices are as timely as possible. Finally, more robust trading of longer-dated SOFR derivatives could facilitate estimation of SOFR term rates of tenors beyond six months, perhaps using a hybrid of the step function approach described here and more traditional parametric specifications such as those described by Nelson & Siegel (1987) and Svensson (1994).

A Model specification and estimation

This appendix provides technical detail on the specification and estimation of our model.

A.1 Data

Estimated term SOFR rates depend on three sources of data. SOFR futures prices provide information about market expectations of overnight SOFR rates in the future. Historical SOFR rates are needed to accurately value those SOFR futures contracts that are currently in their settlement calculation window (i.e., one-month futures contracts with a settlement date less than one month ahead). Finally, FOMC meeting dates are needed to determine when jumps in future SOFR rates are likely to occur. Data are obtained from the following sources.

- CME Group began listing one-month and three-month SOFR futures in May 2018. Daily closing prices for SOFR futures are available directly from CME or through Refinitiv.
- Daily SOFR rates from April 2018 onward are available on the Federal Reserve Bank of New York's website.[7]
- Typically the Federal Reserve Board publishes FOMC meeting dates for the following year in the second quarter of the current year. Thus, it is generally possible to know FOMC announcement dates six months in advance, but over longer periods FOMC announcement dates may not be available. The current FOMC calendar is available on the Federal Reserve Board's website.[8]

A.2 Modeling forward rates

We estimate forward-looking SOFR term rates by leveraging the fact that both futures contract valuations and term rates depend on risk neutral expectations of forward SOFR rates over time. Because forward rates are not directly observable, we begin by specifying a path of forward rates that is a function of a finite-dimensional vector of unknown parameters. We then estimate the function by choosing values for the unknown parameters such that implied futures contract values closely match observed contract prices. Finally, we use the estimated forward rate function to compute term rates for all tenors of interest.

Forward rates are modeled using a step-function specification under which future SOFR rates are assumed to remain constant for all dates between FOMC meetings, but may jump up or down on FOMC policy rate announcement dates. A number of standard parametric functions for forward curves were considered including Nelson-Siegel specifications and spline models. For term rates shorter than one year, we found that a simple step-function specification provided a good fit to observed futures prices, was sufficiently flexible to capture a broad range of possible paths for forward rates, and was empirically tractable given futures price data currently available.

[7] `https://apps.newyorkfed.org/markets/autorates/sofr`
[8] `https://www.federalreserve.gov/monetarypolicy/fomccalendars.htm`

Let t index calendar days. Suppose we wish to estimate forward-looking term rates as of date t_0. In the notation that follows, we will generally omit the as-of date, as it should be clear that all model parameters and input data depend on this date. Denote the forward daily SOFR rate by $f(t)$. Let $M_{(k)}$ be the date of the k-th FOMC announcement that occurs at least one day after t_0. For example, for an as-of date of June 1, 2018, the first FOMC date ($M_{(1)}$) would be June 13, the second ($M_{(2)}$) would be August 1, and so on. Let θ_0 be the forward rate on the day after the as-of date (i.e., t_0+1) and let θ_k be the amount by which the the forward rate changes on the k-th FOMC announcement date. The forward rate at date t is given by

$$f(t;\Theta) = \theta_0 + \sum_k \theta_k 1\{t > M_{(k)}\}, \tag{1}$$

where $1\{\cdot\}$ is the indicator function that returns a value of one if the conditional statement is true and zero otherwise. Since all $M_{(k)}$ are known in advance, the forward rate curve depends on the vector of unknown jump parameters $\Theta = (\theta_0, \ldots, \theta_K)$ where K is the index of the last relevant FOMC date.

A.3 Valuing SOFR futures

A CME SOFR futures price in expiration is equal to $1 - R$ where R is an arithmetic average of observed SOFR rates during the contract month for one-month futures and the compounded daily rate during the reference quarter for three-month futures. Under no-arbitrage conditions, the price of a SOFR future at date t_0 is a function of the forward rate curve and, in some cases, observed SOFR rates.

Let $\mathcal{T}_m$ be the set of calculation dates associated with a one-month contract whose reference month is m months in the future (the current month is $m = 0$) and let N_m^1 be the total number of calendar days in the m-th month. Given the forward rate path defined in equation (1), the implied price as of date t_0 for a one-month contract for delivery in some month $m \geq 1$ is equal to

$$\hat{P}_m^1(\Theta) = 1 - \frac{1}{N_m^1} \sum_{t \in \mathcal{T}_m^1} f(t;\Theta). \tag{2}$$

For contracts expiring in the current month (i.e., $m = 0$), the price depends on the observed SOFR rates on or before the as-of date and the forward rates for dates between the as-of date and the expiry date. Let r_t denote the SOFR rate reported on date t or the last business day before t if t falls on a weekend or holiday. Let $\mathcal{T}_0^{1+} = \{t \in \mathcal{T}_0^1 | t > t_0\}$ be the set of calculation dates occurring after the as-of date, and let $\mathcal{T}_0^{1-} = \{t \in \mathcal{T}_0^1 | t \leq t_0\}$ be the set of dates occurring on or before the as-of date. The implied price for the one-month contract expiring in the current month is

$$\hat{P}_0^1(\Theta) = 1 - \frac{1}{N_0^1} \left(\sum_{t \in \mathcal{T}_0^{1-}} r_t + \sum_{t \in \mathcal{T}_0^{1+}} f(t;\Theta) \right). \tag{3}$$

Imputed prices for three-month contracts are computed in a similar manner, but compounded

rates are used and holidays and weekends are treated differently from business days. Let $\tilde{\mathcal{T}}_q^3$ be the set of calculation *business* calendar dates associated with a three-month contract whose reference quarter is q quarters in the future (the current reference quarter is $q = 0$). Let N_q^3 be the number of calendar days in the q-th quarter and let d_t be the number of calendar days from date t to the next business day. The implied price for a three-month contract delivered in a future quarter is

$$\hat{P}_q^3(\Theta) = 1 - \frac{360}{N_q^3}\left(\prod_{t\in\tilde{\mathcal{T}}_q^3}\left(1 + \frac{f(t;\Theta)d_t}{360}\right) - 1\right). \tag{4}$$

As with one-month contracts, the price of three-month contracts expiring in the current quarter (i.e., $q = 0$) depends on both observed and forward SOFR rates. Let $\tilde{\mathcal{T}}_0^{3+} = \{t \in \tilde{\mathcal{T}}_0^3 | t > t_0\}$ be the set of business days occurring after the as-of date and let $\tilde{\mathcal{T}}_0^{3-} = \{t \in \tilde{\mathcal{T}}_0^3 | t \leq t_0\}$ be the set of business days occurring on or before the as-of date.

$$\hat{P}_0^3(\Theta) = 1 - \frac{360}{N_0^3}\left(\prod_{t\in\tilde{\mathcal{T}}_0^{3-}}\left(1 + \frac{r_t d_t}{360}\right)\prod_{t\in\tilde{\mathcal{T}}_0^{3+}}\left(1 + \frac{f(t;\Theta)d_t}{360}\right) - 1\right). \tag{5}$$

A.4 Estimation

Θ is estimated by choosing values for the initial forward rate and unknown jump parameters that minimize deviations between observed futures prices and those implied by equations (2) through (5). To ensure that we are able to compute term rates out to six months, we use futures prices for all seven outstanding one-month contracts and the three most current three-month contracts.

Depending on how the calculation periods for available futures contracts line up with FOMC announcement dates, some elements of Θ may not be well identified. To address this potential issue, we impose two constraints on the model.

1. *Policy Gradualism.* If two or more FOMC meetings fall withing the period covered by a single three-month contract, then multiple jump patterns for forward rates may lead to the same implied contract prices. For example, if a three-month contract price indicates that forward rates should rise during a quarter containing two FOMC announcement dates, then a large increase at the first meeting and no increase at the second meeting, or a large increase at the second meeting and no increase at the first meeting, or equal-sized increases at both meetings may all be consistent with the same contract price. To pin down the jump pattern in these types of situations, we assume that wherever multiple jump patterns are possible the FOMC will always choose the path that minimizes the absolute size of the largest individual jump. In the example above, this would imply two equal-sized policy rate increases during the quarter.

2. *Six Month Jump Window.* We assume that no jumps will occur more than six months after the as-of date. This assumption obviates the need to make assumptions about the timing of distant FOMC meetings that may not yet have been scheduled and reduces the sensitivity of longer term rates to prices of far-dated futures contracts where trading is thin. This assumption is

needed even if term rate tenors do not exceed six months because the calculation periods for relevant futures contracts may exceed the tenor of the term rates.

Denote observed prices for one- and three-month contracts by P_m^1 and P_q^3 respectively. Our estimator, $\hat{\Theta}$, solves the following optimization problem:

$$\min_{\Theta} \left(\sum_{m=0}^{6} w_m^1 \left(P_m^1 - \hat{P}_m^1(\Theta) \right)^2 + \sum_{q=0}^{2} w_q^3 \left(P_q^3 - \hat{P}_q^3(\Theta) \right)^2 \right)^{\frac{1}{2}} + \lambda \left(\sum_k (\theta_k)^2 \right)^{\frac{1}{2}}. \tag{6}$$

The first term is the mean squared errors for imputed valuations for one- and three-month contracts. The second term is a penalty function that imposes the assumption of policy gradualism. $\lambda > 0$ is a weight parameter set to be so small that it does not materially affect the parameter estimate unless one or more elements of Θ are not identified from observed contract prices. w_m^1 and w_q^3 are weighting parameters that allow one to put greater weight in the objective function on those contracts whose prices are believed to be more accurate. For example, one could place greater weight on contracts with higher average trading volume or more timely price quotes. In this analysis, we weight all contracts equally.

A.5 Term rates

Forward term rates derived from equation (1) are reported each business day for terms of one, three, and six months. Rates are constructed using conventions similar to, but not identical to, those used for USD LIBOR. Compounded returns are expressed as actual/360. We adopt the OIS compounding convention whereby returns over weekends and holidays are not compounded. A term rate's first accrual date begins on the next business day following the rate's as-of date. Let $\tilde{\mathcal{T}}(T)$ be the set of business calendar days from the first accrual date to a date T days in the future.[9] The forward-looking compounded SOFR rate (annualized) for financing over a period of T days is

$$h(T) = \frac{360}{T} \left(\prod_{t \in \tilde{\mathcal{T}}(T)} \left(1 + \frac{f\left(t; \hat{\Theta}\right) d_t}{360} \right) - 1 \right). \tag{7}$$

Term periods are set using the modified following business day convention. We begin by choosing an initial end-of-term date that falls on the same calendar day as the first accrual date one, three, or six months in the future. If that date lands on a holiday or weekend, the end-of-term date rolls forward to the next business day unless that date falls in a different calendar month, in which case the end-of-term date rolls backward to the immediately preceding business day.

[9] We use the NASDAQ business day calendar, which is slightly different from that used for USD LIBOR.

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Table 1: Spreads between estimated forward-looking term SOFR and federal funds OIS in basis points.

Term	Spread	Mean	Median	Standard Deviation	5th Percentile	95th Percentile
Overnight	SOFR-EFFR	2.5	1.0	8.8	-3.0	11.0
1 Month	Term SOFR-OIS	1.6	1.4	1.4	-0.4	4.1
3 Month	Term SOFR-OIS	1.7	1.7	1.4	-0.3	4.0
6 Month	Term SOFR-OIS	1.6	1.5	1.6	-0.8	4.3

Table 2: Spreads between federal funds futures implied term rates and realized compounded federal funds rates in basis points.

Term	Mean	Median	Standard Deviation	5th Percentile	95th Percentile
A: Full sample (Jan. 1, 2000 – Jan. 22, 2019)					
1 Month	1.3	0.1	8.0	-4.5	7.7
3 Month	4.4	0.6	15.6	-6.0	30.7
6 Month	10.5	1.6	28.3	-13.0	86.8
B: Pre-crisis (Jan. 1, 2000 – Jun. 30, 2007)					
1 Month	0.6	-0.3	7.0	-5.8	8.3
3 Month	3.9	-0.2	14.0	-9.3	33.5
6 Month	11.6	0.8	28.8	-16.6	78.8
C: Crisis (Jul. 1, 2007 – Dec. 31, 2010)					
1 Month	5.5	1.5	14.9	-6.1	37.9
3 Month	14.8	4.4	26.8	-2.2	83.6
6 Month	31.0	9.0	42.1	-0.9	126.5
D: Post-crisis (Jan. 1, 2011 – Jan. 22, 2019)					
1 Month	0.2	0.0	1.2	-1.7	2.2
3 Month	0.1	0.4	2.6	-3.7	3.7
6 Month	-0.0	0.5	4.7	-8.3	6.4

Table 3: Spreads between term rates based on federal funds futures and federal funds OIS in basis points.

Term	Mean	Median	Standard Deviation	5th Percentile	95th Percentile
A: Full sample (Jan. 1, 2000 – Jan. 22, 2019)					
1 Month	-1.5	-0.0	12.1	-11.7	5.5
3 Month	-0.2	-0.1	1.9	-2.4	1.7
6 Month	-0.0	-0.0	1.4	-1.8	1.9
B: Pre-crisis (Jan. 1, 2000 – Jun. 30, 2007)					
1 Month	1.1	1.2	3.6	-4.4	6.8
3 Month	-0.4	-0.3	1.9	-3.2	2.3
6 Month	-0.0	-0.1	1.6	-2.4	2.5
C: Crisis (Jul. 1, 2007 – Dec. 31, 2010)					
1 Month	-10.4	-1.0	26.0	-56.1	12.1
3 Month	0.2	0.1	3.4	-2.4	3.7
6 Month	0.0	0.1	2.3	-2.0	2.5
D: Post-crisis (Jan. 1, 2011 – Jan. 22, 2019)					
1 Month	-0.1	-0.1	0.6	-1.2	0.5
3 Month	-0.1	-0.1	0.3	-0.6	0.3
6 Month	-0.0	-0.1	0.2	-0.5	0.4

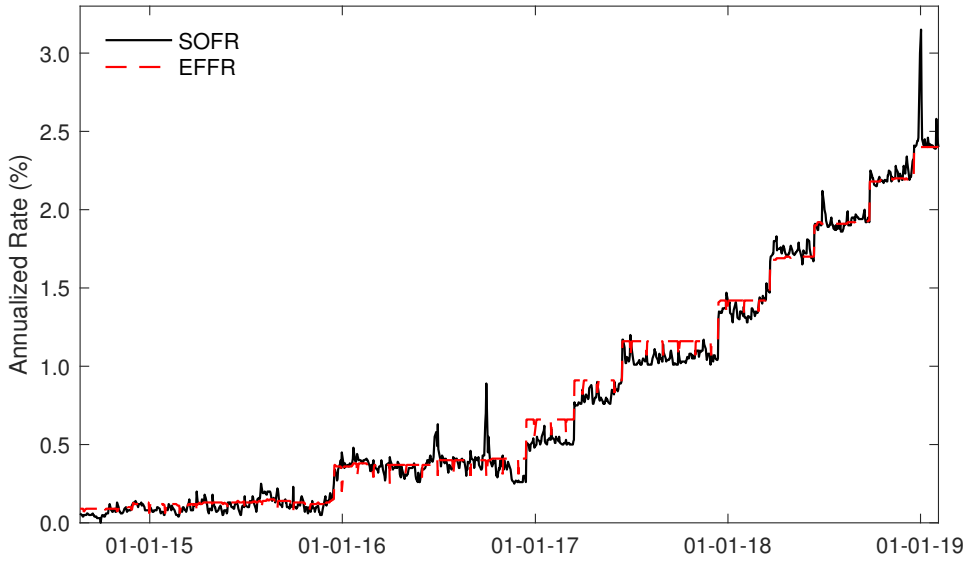


Figure 1: Secured Overnight Financing Rate and Effective Federal Funds Rate over time.
Source: Federal Reserve Board, Federal Reserve Bank of New York.

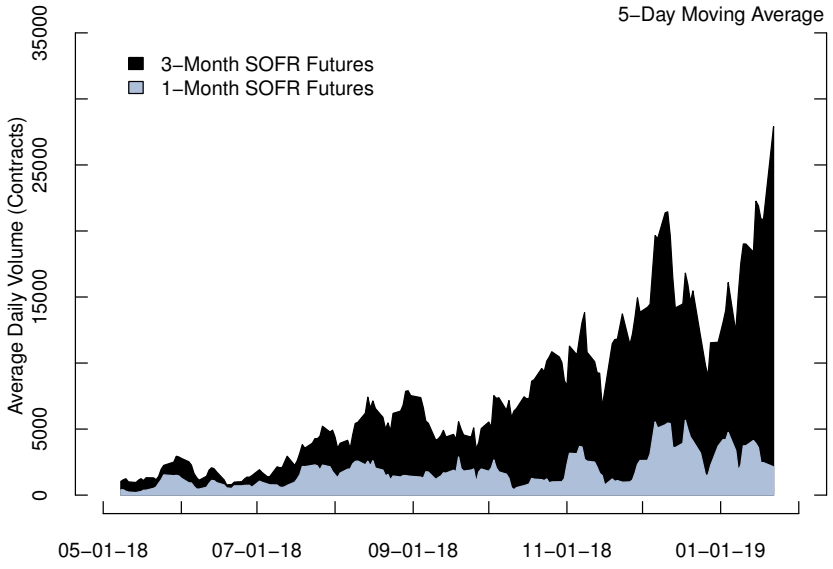
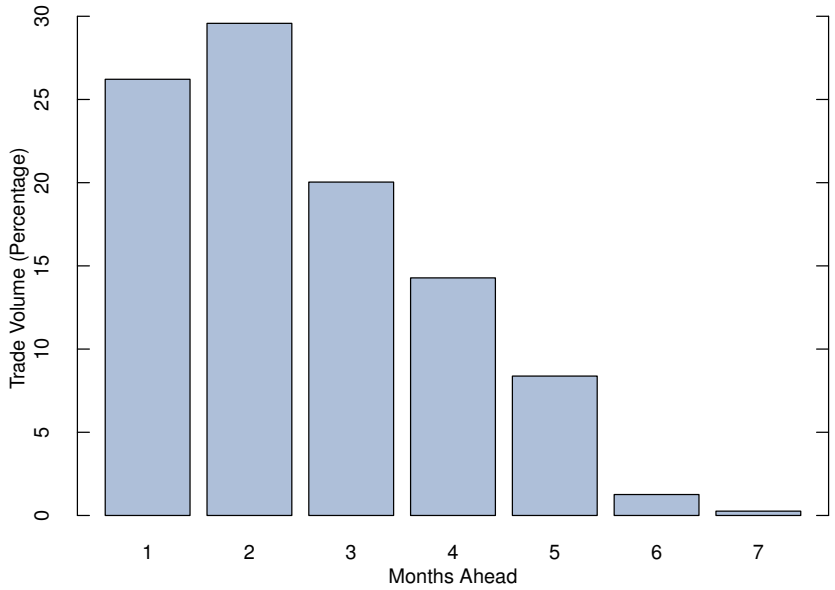


Figure 2: Average daily trading volume in CME SOFR futures contracts.
Source: Authors' calculations using Refinitiv data.

One-Month Contracts



Three-Month Contracts

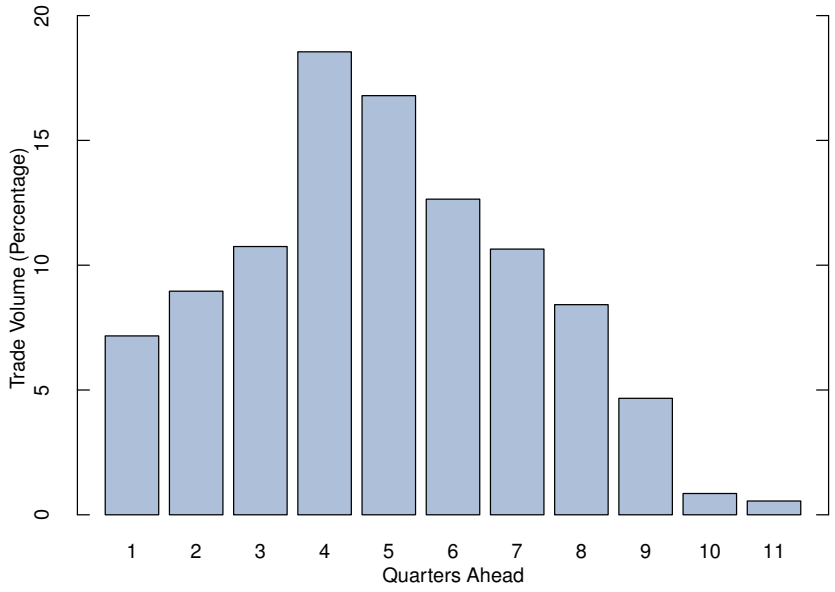


Figure 3: Distribution of CME SOFR futures contract trading volume.
Source: Authors' calculations using Refinitiv data.

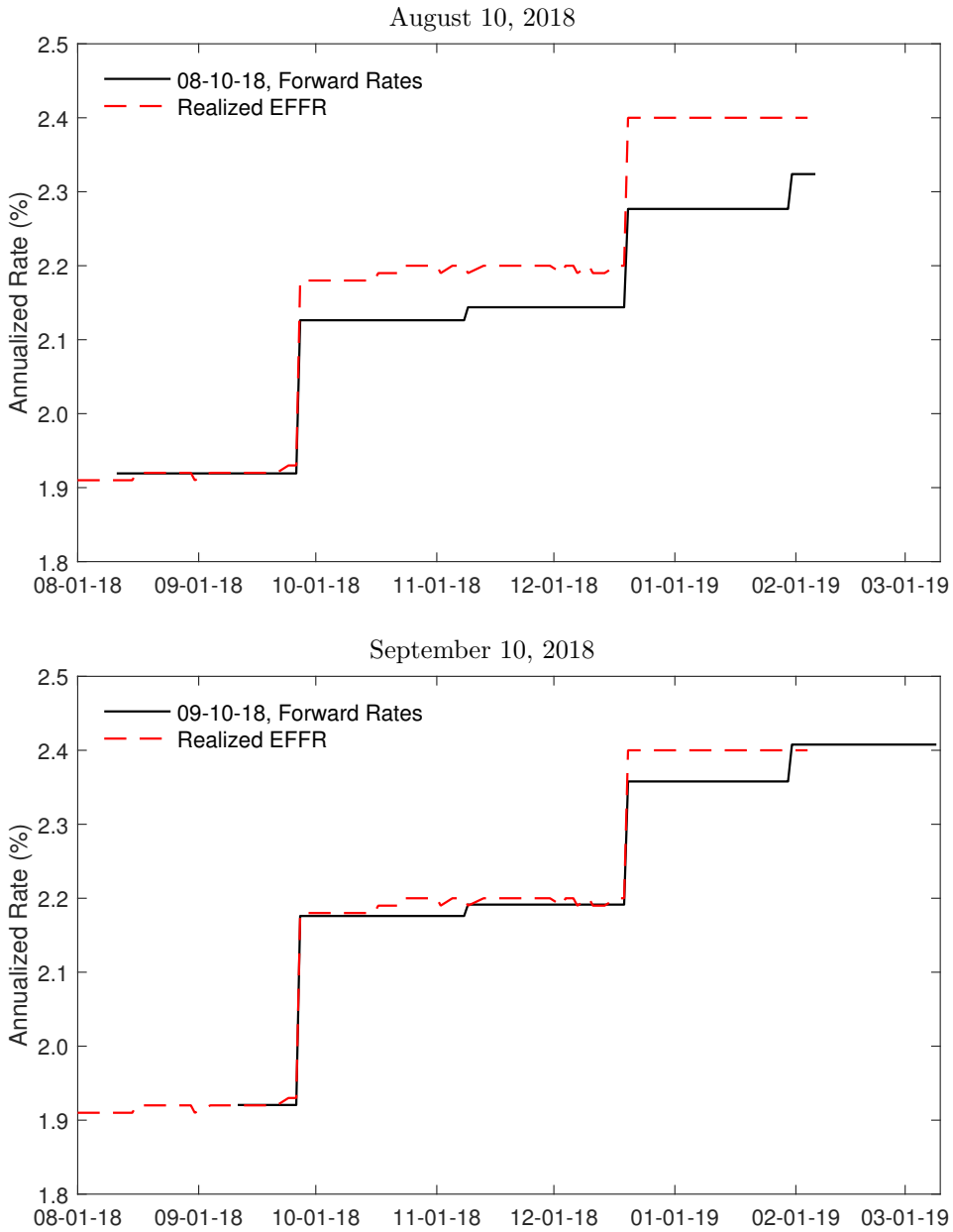


Figure 4: Estimated forward SOFR rates for selected dates.
Source: Federal Reserve Board and authors' calculations using Rifinitiv and Federal Reserve Bank of New York data.

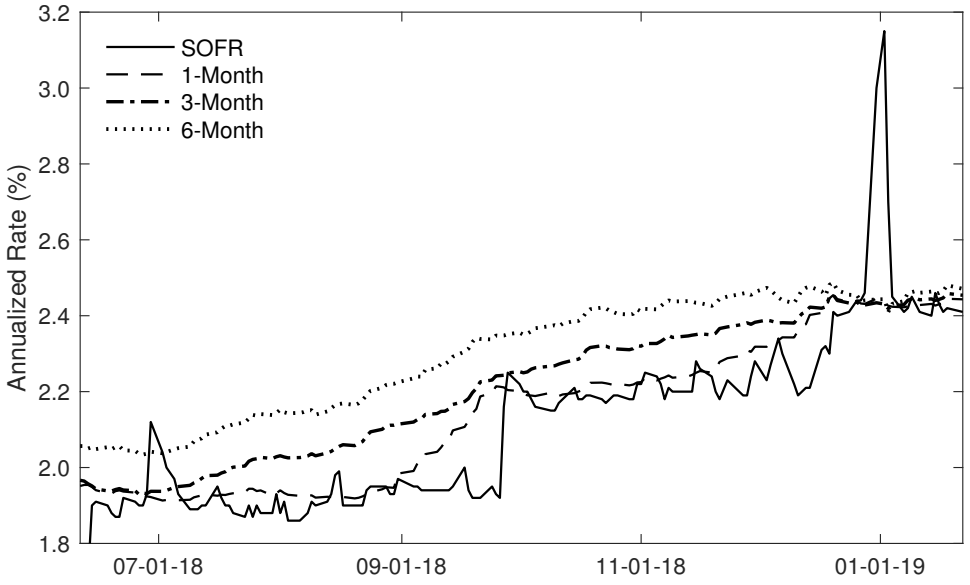


Figure 5: Term SOFR rates over time.
Source: Authors' calculations using Rifinitiv and Federal Reserve Bank of New York data.

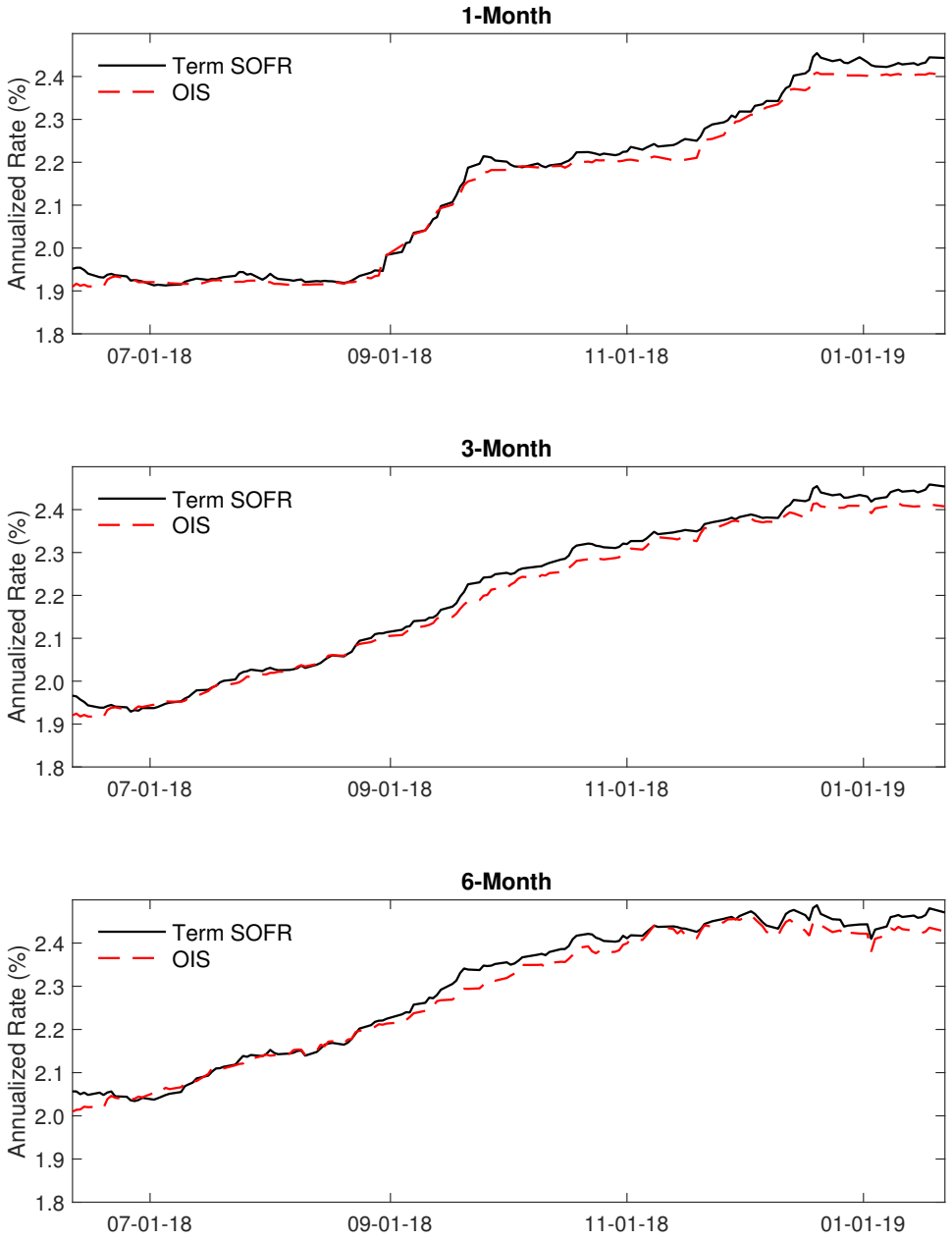


Figure 6: Term SOFR and federal funds OIS rates over time.
Source: Authors' calculations using Rifinitiv and Federal Reserve Bank of New York data.

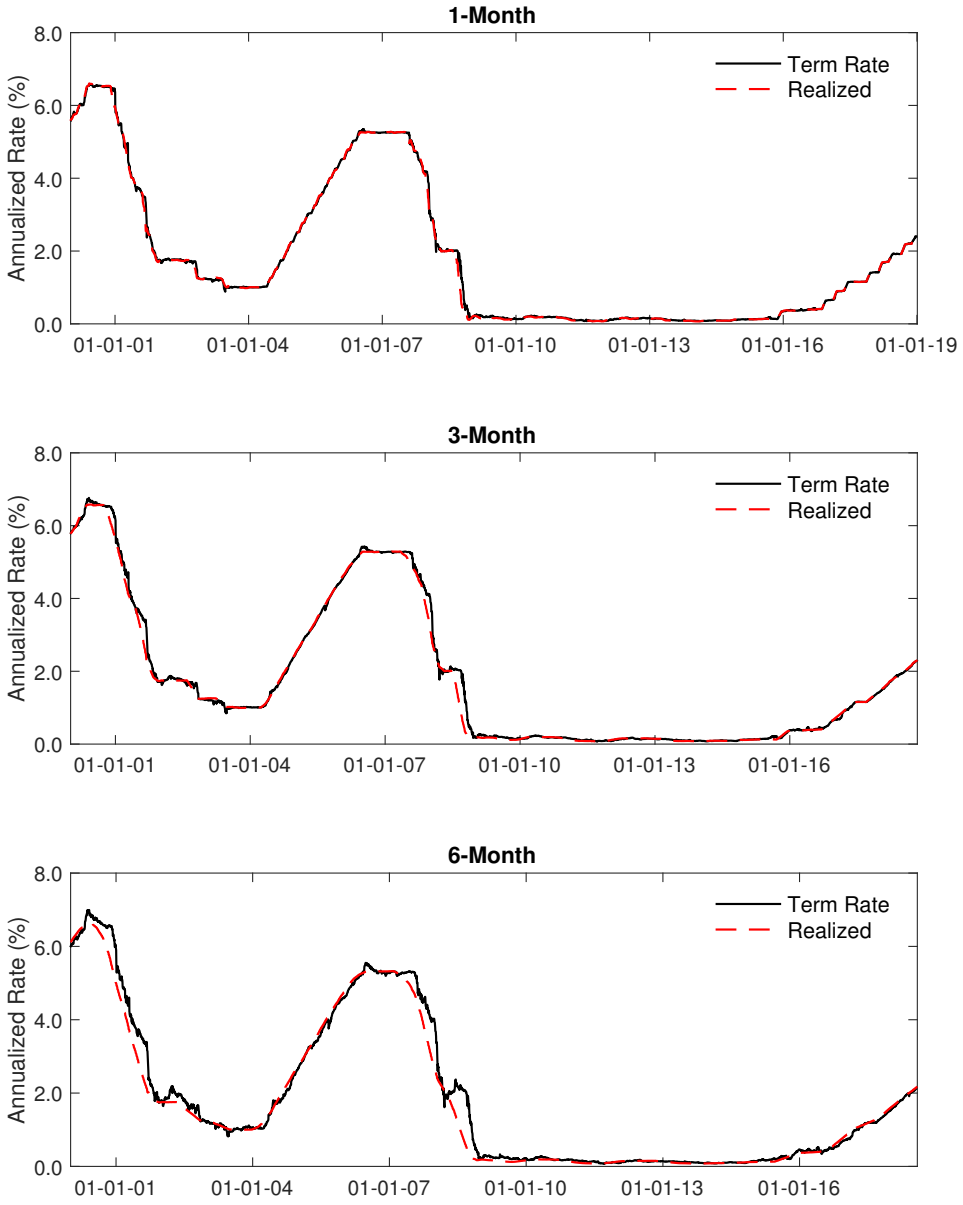


Figure 7: Federal funds futures implied term rates and realized compounded federal funds rates over time.
Source: Authors' calculations using Rifinitiv and Federal Reserve Bank of New York data.

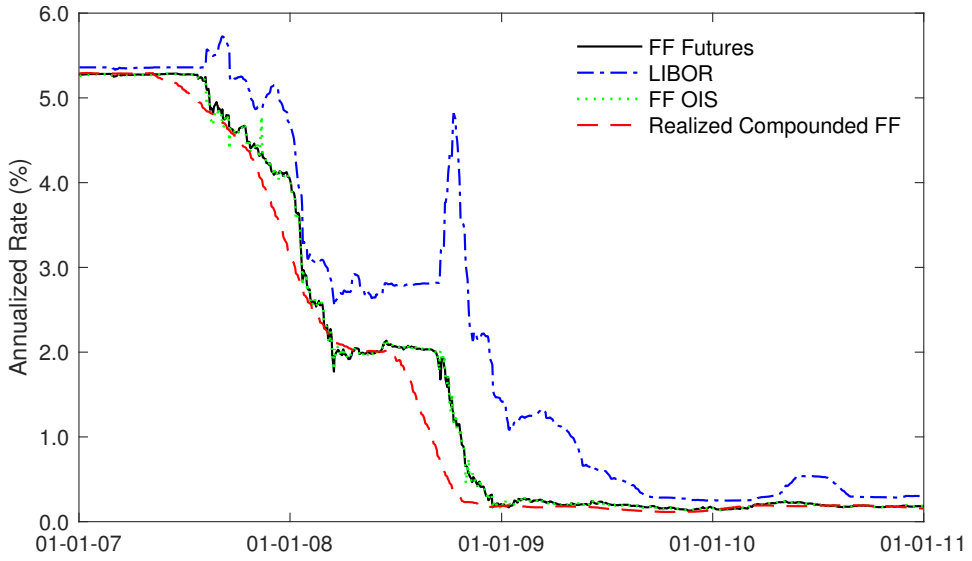


Figure 8: Selected three-month forward-looking term rates and three-month compounded federal funds rate during the financial crisis.
Source: Authors' calculations using Rifinitiv and Federal Reserve Bank of New York data.

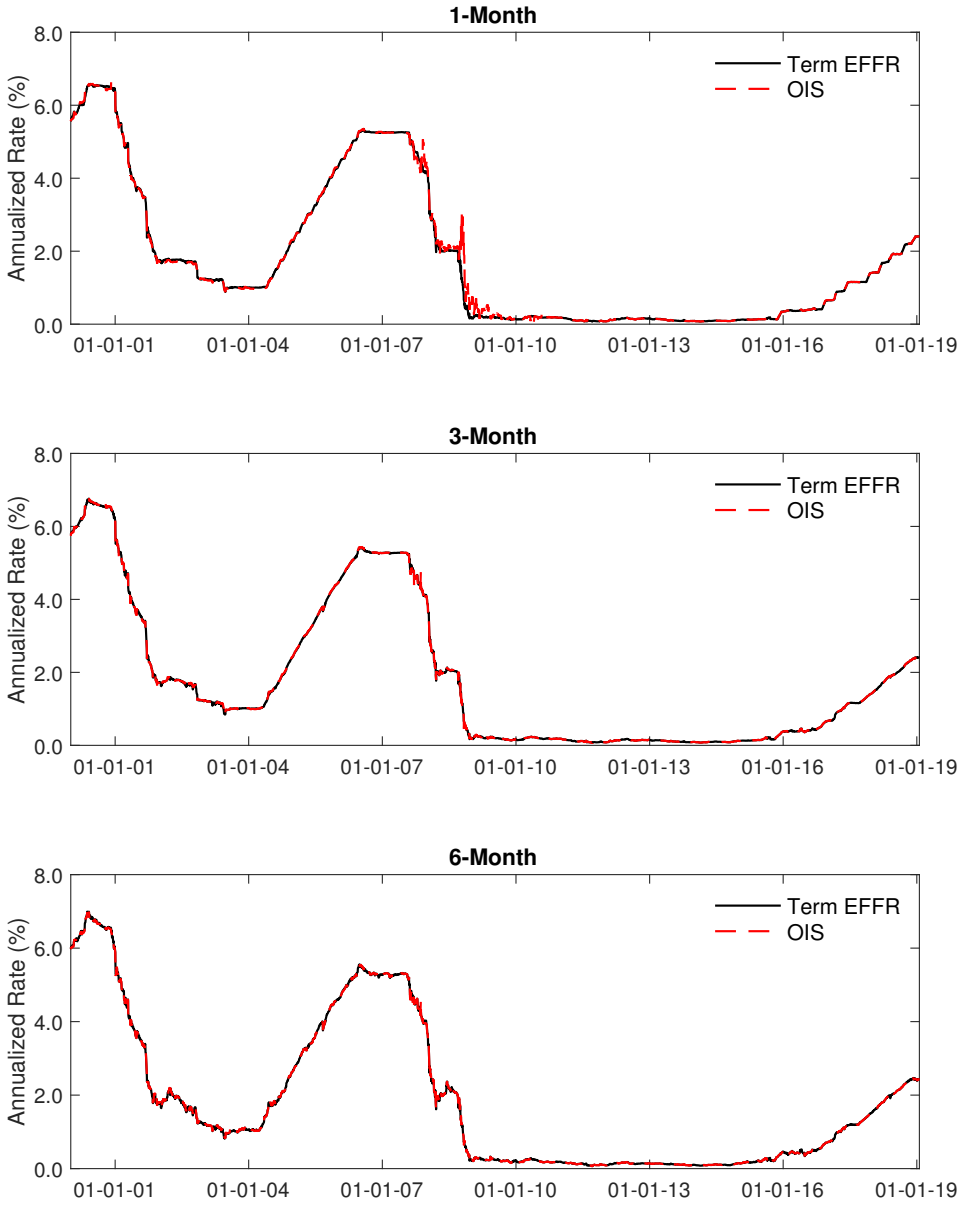


Figure 9: Term rates based on federal funds futures and federal funds OIS over time.
Source: Authors' calculations using Rifinitiv and Federal Reserve Bank of New York data.

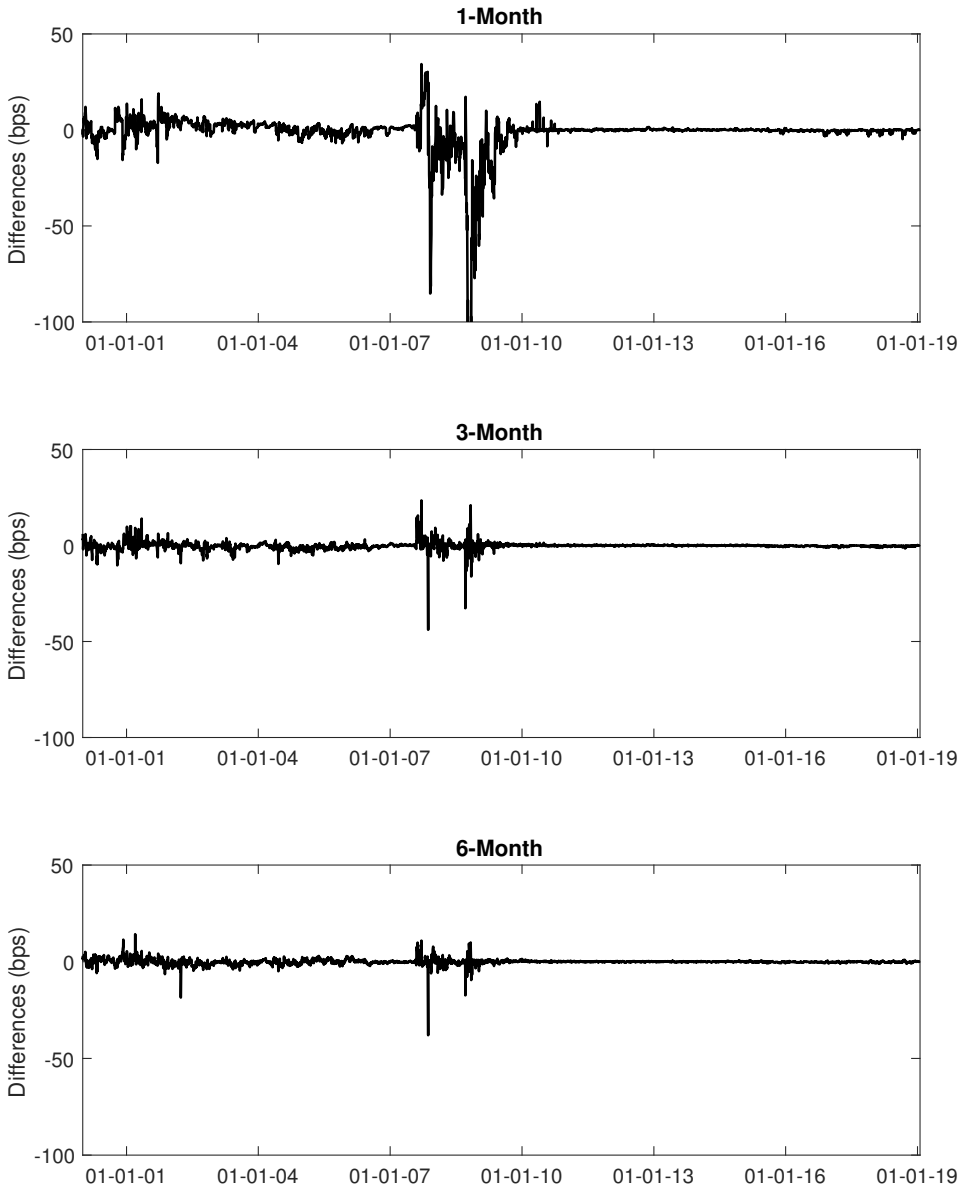


Figure 10: Spreads between term rates based on federal funds futures and federal funds OIS over time.
Source: Authors' calculations using Rifinitiv and Federal Reserve Bank of New York data.